Designing Rules for the Capacity Market

Hélène Le Cadre, Michaël Soubra

Les Cahiers de la Chaire

Chaire Modélisation prospective au service du développement durable

Designing Rules for the Capacity Market

Hélène Le Cadre et Michaël Soubra

Octobre 2013

Working Paper N° 2013-03-10

Designing Rules for the Capacity Market

Hélène Le Cadre **Michaël Soubra**
MINES ParisTech,
Centre for Applied Mathematics,
Sophia Antipolis, France
first name.last name@mines-paristech.fr

Abstract

In this article, the energy market is a modeled as a Stackelberg game involving three categories of agents: microgrids made of end users sharing the same energy provider, suppliers and generators. The energy production is decentralized involving non-renewables, renewables and demand response performed at the microgrid level. We compute analytically the Nash equilibrium of the game in the generators' production prices, efforts invested in their capacity, and, in the suppliers' energy orders. Furthermore, we prove that the generators' and the demand response prices can be obtained as functions of the price paid by the end users per unit of demand. Finally, coupling the energy and the capacity markets, we design rules for the capacity market guaranteeing the system wide balance and the market opening to new investors while avoiding moral hazard and abuse of dominant positions.

1 Introduction

Capacity markets have proven to be one of the most contentious elements of electricity restructuring [2], [3], [7], [14]. Many argue there is no need for a capacity market [14]. Other argue that, while they may be needed, the current designs are inadequate [4]. Still other argue that capacity markets are essential for encouraging sufficient investment in new capacities. The relative reluctance for the introduction of a capacity market is partly due to the failures observed in the US where investments were insufficient and market power occured [5], [6], [10], [35]. The Nome law [36], voted by the French government in november 2010, aims at accelerating the liberalization of the energy market and imposes the creation of a capacity market in 2015 which long term goal should be to balance the offer and the demand. This decision follows the Sido-Poignant report [37] which puts forward the alarming growth of electric consumption peaks in France. The necessity of such a law can be justified by the observation that the market does not seem to pay enough for the investments in peak capacities i.e., peak productions and voluntary demand responses. Since the market remunerates exclusively the produced energy volumes, the investments in peak capacities are insufficient. This can lead, in case of a lack of capacity, to the electrical network blackout, unbearable for the consumers and generating an exorbitant cost for the suppliers [19], [25]. A capacity market is based on the principle that it should provide the guarantee that the electrical system has enough capacity to satisfy the consumers' needs during the peaks of consumption, either by requiring suppliers to provide the evidence that they have enough capacity or, by elaborating a supplementary mechanism involving the regulator's intervention to guarantee the balance between the demand and the supply [26]. This supplementary mechanism is called feedback mechanism in the capacity market literature. The role of regulator or market designer can be played indifferently by the network operator or by the State. In the article, the regulator will be called Principal. Indeed, this is the dedicated term used in game theory to designate the agent who centralizes the information and implements the mechanism optimizing the system wide operations [23], [24].

Delivering a reliable power supply to consumers has always been a central objective of market design and various solutions to this challenge have been adopted all over the world. This diversity reflects the differences in power systems characteristics, including the mix of resources[1] used to generate electricity and to balance the supply and the demand [20], [27], [34]. The EU commission analysis confirms that the share of power generation provided by renewables will need to continue to increase after 2020 in all scenarios considered, exceeding 50% in 2030. Resource availability within Europe implies that a significant proportion of this renewable generation will have to be produced from resources that are only intermittently available such as solar, wind, swell, etc. This rise raises a new type of reliability question regarding the definition of the right type of resources in which investments should be concentrated [4], [9]. This is described in the literature as the system quality challenge [9]: how can the Principal incentivate the generators to invest in sufficient capacities in the right locations and of the right type i.e., according to the most economically efficient mix [19], and also in demand response, in storage resources and processes, to satisfy a reliability standard at least cost [4], [9]? Both short term and long term aspects should be differentiated in the system quality. In the short term the question is whether the system has effective access to all of the cost effective flexibility[2] available from the existing resource portfolio including existing demand response potential [14]. The long term aspect is whether the market supports investment in a portfolio of new and existing supply and demand side resources capable of efficiently and cost effectively meeting the projected need for flexible resource capabilities over investment time horizons. The power system will need resources capable of rapidly changing output or switching demand frequently and continuously throughout the year depending on the energy availability from intermittent renewables [33]. Besides, the balancing mechanism whereby the system administrator buys and sells energy in real time to maintain the system balance has become a critical element of power market design. Its expanding through physical interconnections between regions is necessary to decrease the probability of extreme events [33].

Capacity and energy markets are clearly coupled. In this article, this latter is modeled as a Stackelberg game. Such an approach was successfully used in [15], [16] to capture the hierarchical economic structure of the smart grid. Three categories of ac-

[1]Over time, existing resources will become uneconomic, often as a result of changes in environmental regulations or the cost of carbon emissions, and will be closed.

[2]The essential parameters to define the flexibility are how fast, how far and how frequently a resource can be started and stopped or ramped up and down both within scheduling intervals and across multiple scheduling intervals.

tors are considered: the suppliers who deliver energy to the end users, the microgrids which are aggregations of end users sharing a common geographic area and the same provider, the generators who hold capacities and produce energy. The microgrids can become producers. This point will be detailed later in the article. The market structure i.e., the number of generators and the number of suppliers, are considered as parameters of the game. At the lower level, the suppliers optimize the quantities of energy that they buy to the generators and to the microgrids in order to maximize their utility. At the upper level, the generators optimize their price per energy unit and their effort so as to maximize their utility. A second market occurs between the Principal, which can be indifferently the regulator, the State, etc., and the capacity owners. The capacity owners have the opportunity to perform an effort, by investing in their source of energy, to reduce their marginal cost. In turn, this enables them to produce more at the same cost. The major difficulty is that the production generation process is decentralized, mainly due to the introduction of renewable energy sources, and very difficult to forecast. To balance the supply and the demand, to avoid moral hazard [11], [12], [23], to guarantee the market opening to new investors and to avoid abuse of dominant positions, the Principal aims at designing rules for the game based on side-payments transferred to the generators and on the delivery of capacity certificates. The capacity market mechanism is designed under the assumption that the demand reaches a peak of consumption plus a reservation margin to prevent from extreme events.

We detail the article organization. In Section 2, we describe and solve the Stackelberg game occuring between the suppliers and the generators. In Section 3, we design the mechanism checking the four properties above mentioned. In Section 4, we run simulations enabling us to calibrate the market parameters and to check the efficiency of the designed mechanism.

2 The energy market model

We consider a holistic system made of three imbricated levels, as depicted in Figure 1.

In the first level, the $K \in \mathbb{N}^*$ generators $(G_k)_{k=1,\dots,K}$ capacities of production are determined according to the processes $(\bar{q}_k)_{k=1,\dots,K}$. For example, we can make the assumption that the producers are classified into categories of resources such as: nuclear, gas, coal, oil, hydraulic, wind, solar. Each generator produces a specific category of resource. This aggregation hypothesis reduces the uncertainty on the renewable energy production. The generators can either open or shut down their production capacities. The state of the production capacity associated with generator G_k i.e., either open or closed, is contained in a variable $a_k \in \{0;1\}$ such that $a_k = 1$ if the production capacity is open and $a_k = 0$ if it is closed. The generators optimize independently and simultaneously their energy price and their effort, also called investment [1], [30], so as to maximize their utility. This latter coincides with the revenue resulting from the selling of their resource production to the suppliers minus the cost and the effort dedicated to their production. In the model, the effort can be thought in terms of a generator renewing equipments to more efficient ones, expanding production capacities, adopting improved technologies, or investing in research and development for process innovation [30].

In the second level, the $N \in \mathbb{N}^*$ suppliers $(s_i)_{i=1,\ldots,N}$ buy energy to satisfy their microgrid energy demand. Throughout the paper, we will assume that microgrid M_i's energy demand, D_i, is the value reached at peaks of consumption, as publicly declared by the suppliers, plus a reservation margin, which takes into account the uncertainties and the biases in the estimation resulting from extreme events. Each supplier s_i provides energy to a single microgrid M_i. This assumption can be justified by geographic considerations: the cost of energy transport being high [22], each supplier covers a bounded geographic area. There is no overlapping between the geographic areas. We let q_{ik} be the quantity of energy[3] bought by supplier s_i to generator G_k. Supplier s_i can obtain energy through three means: they can buy energy to the generators or, to the microgrids, provided these latter perform demand response or, they can exchange energy with the other suppliers. The microgrid capacities of production will be determined according to the processes $(\bar{q}_i^e)_{i=1,\ldots,N}$. The capacities bought by the supplier should be carefully optimized, so as to guarantee that his microgrid will be served during the peaks of consumption. The suppliers optimize independently and simultaneously the bought quantities of energy so as to maximize their utility. This latter coincides with the difference between, on the one hand, the sum of their net utility representing their preferences regarding the energy sources, which are symmetrically related substitutes, and of the price paid by the microgrid to meet its demand, and, on the other hand, the costs of the energy purchases to the portfolio of generators.

In the third level, microgrids contain eco-neighborhoods made of houses, of various categories of firms and of non-profit State administered organizations, etc. [17]. The microgrids can become generators by deploying solar pannels or wind turbines either at individual scales or in commonly shared areas [29]. They can also perform storage thanks to electric vehicle batteries, chemical or thermal processes [17]. Demand responses can be performed at the consumer level [22]. It enables the shifting of the consumer demand from one period where a peak of consumption is expected, to another period where the demand remains low [32]. It can be implemented in two ways: either through contracts defined between the suppliers and the end users or through pricing mechanisms providing economic incentives for the end users to shift their consumption. Some microgrids might produce more than their current demand generating energy surpluses. Microgrids, which produce not enough to meet their own demand, might bargain with microgrids generating energy surpluses [29]. An energy market might therefore occur between the microgrids. However, in our model, the exchanges of energy between the microgrids would be performed by the suppliers. As already mentioned, the cost of energy transport being high, it is unrealistic to assume that exchanges occurred between geographically distant suppliers. Furthermore, we assumed that each supplier delivers energy to a well delimitated geographic area and that there is no overlapping. As a result, the geographic provision areas being wide enough, we choose to ignore the energy transfers between the suppliers because they would be negligeable.

Other agents such as a regulator or the State can intervene. They will be aggregated in a single agent called Principal. His objective is to induce supply to invest in

[3]The quantities of energy bought by the supplier to each generator coincides with a part of his production capacity. In fact, the suppliers buy capacity access to the generators. In return, the generators undertake to release these capacities when peaks of consumption occur.

sufficient generation in the right locations and of the right type i.e., according to the most economically efficient mix [19], and also in demand side and in storage resources [9], to satisfy a reliability standard at least cost [4], [9]. Of course, the Principal may promote certain categories of energy compared to others according to some ideological positions like green (i.e., renewables) or clean (i.e., without carbon emission) versus non-green, political reasons, etc. [22].

We now introduce the following definitions with the resource use:

Definition 1.
- *The number of resources among the K which are active is contained in $\bar{K} \in \mathbb{N}^*$ such that $0 \leq \bar{K} \leq K$.*
- *For resources $\bar{K}+1 \leq k \leq K$ that are inactive, we use the conventions: $q_{ik} = 0, \forall i = 1, ..., N$ and $\bar{q}_k = 0$.*
- *Resource $k = 1, ..., \bar{K}$ is said to be saturated if, and only if, $\sum_{i=1,\ldots,N} q_{ik} = \bar{q}_k$.*
- *The number of resources among the $\bar{K}$ which are saturated is contained in $K^s \in \mathbb{N}^*$ such that $0 \leq K^s \leq \bar{K}$.*

According to the literature, guaranteeing the electrical system reliability can be performed using two approaches: either requiring the suppliers to provide the evidence that they have enough capacity or elaborating a supplementary mechanism involving the Principal's intervention to guarantee the network balance [26]. This latter is known as feedback mechanism in the capacity market literature [7], [20], [26]. The Principal's intervention is captured through side-payments addressed to the generators. It can take the form of punishments such as carbon taxes when the Principal's objective is to minimize the system wide ecological footprint, or, on the contrary, of financial supports in case where the Principal wants to invest in a specific category of energy[4].

2.1 Description of the generators' optimization program

Generator G_k determines his price $\tilde{p}_k$ per unit of energy bought by the suppliers and his effort u_k , so as to maximize his utility. The generators' prices are supposed ordered so that:

$$\tilde{p}_{k-1} \leq \tilde{p}_k \leq \tilde{p}_{k+1}, \ \forall k = 2, ..., \bar{K}-1 \tag{1}$$

and at the boundaries: $0 \leq \tilde{p}_1 \leq \tilde{p}_2$ and $\tilde{p}_{\bar{K}-1} \leq \tilde{p}_{\bar{K}} < +\infty$. This order is arbitrarily chosen. Following the merit order principle [22], the suppliers will start to buy energy to the cheapest generator until his resource becomes saturated then, they will buy energy to the second cheapest, and so on until satisfaction of the microgrid whole demand.

Generator G_k's utility is:

$$\tilde{\pi}_k = (\tilde{p}_k - c_k) \sum_{i=1,\ldots,N} q_{ik} - u_k + \mathcal{T}_k \tag{2}$$

[4]Examples of financial supports provided by the State can be found in the deployment of renewables, in power plant dismantling or, in the investment in research and development efforts promoting fourth generation reactors.

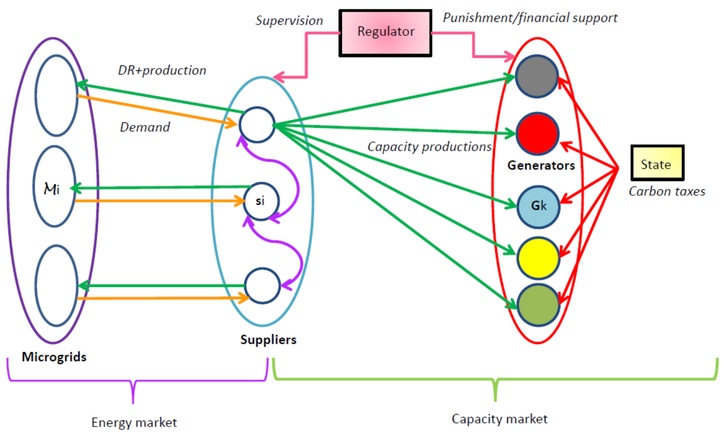


Figure 1: A holistic system.

The marginal cost c_k, supported by generator G_k, decreases exponentially in the effort made by the generator. The dependency is captured by the following equation:

$$c_k = c_{k0} \exp\Big(- \rho\psi(u_k)\Big) \tag{3}$$

and the set of initial conditions $\{c_{k0}\}_{k=1,\ldots,\bar{K}}$. The parameter $\rho > 0$ denotes the potential technological progress characteristic to the industry, also referred as technological opportunity, which can be attained if generators make what the literature calls in general terms an effort [30]. In the model it is denoted $u_k \geq 0$. The form of Equation (3) captures economies of scales which are highly prevalent in industries based on network infrastructures or involving large investments in research and development. Here, they are all the more important as the capacities are aggregated by resource category. Besides, the form of the equation implies that each generator's effort impacts only his own cost and not his competitors' i.e., there is no spillovers or no leakage of information between the generators which would have led to cross-effects in their cost reduction. The function $\psi(.)$ is a measure of the efficiency of the effort in reducing costs. We impose natural properties to the efficiency function: it must be increasing with the effort $\psi'(.) > 0$ but decreasing returns to scale $\psi''(.) < 0$, zero if no effort is made $\psi(0) = 0$. $\mathcal{T}_k \leq 0$ contains the side-payments between the Principal and generator G_k. It is negative in case of punishments resulting from the network imbalance.

For any generator $G_k,\ k = 1, ..., K$, the optimization program takes the form:

$$\begin{aligned} \max_{\tilde{p}_k, u_k} \quad & \tilde{\pi}_k \\ \text{s. t.} \quad & \tilde{p}_{k-1} \leq \tilde{p}_k \leq \tilde{p}_{k+1} \\ & u_k \geq 0 \end{aligned}$$

2.2 Description of the suppliers' optimization program

Supplier s_i determines the quantity of resource q_{ik} and e_i that he buys to generator G_k and to microgrid M_i respectively, so as to maximize his utility. D_i will represent the end users' demand during the peaks of consumption plus a reservation margin. The end users in the microgrid M_i pay supplier s_i, p_i per unit of D_i. Depending on the form of the implemented contract, supplier s_i has the opportunity to complement the quantities of energy that he bought to the generators by buying the missing quantities to the end users. These latter can release energy through demand responses, through their own production or through storage devices. The sum of the energy released by the microgrid M_i through demand responses, various means of productions and storage, is stored in $\bar{q}_i^e$. Microgrid M_i is retributed by supplier s_i according to the unit price p_i^e.

Various approaches to model the product preferences exist in the micro-economic literature [21], [31]. The K resources categories are symmetrically related substitute. In our model, the representative supplier s_i has preferences following Shubik and Levitan's quadratic utility function [8], [30][5]. His net utility takes the form:

$$\begin{aligned} U_i &= \sum_{k=1,\dots,\bar{K}} q_{ik} + e_i - \frac{1}{2}\Big(\sum_{k=1,\dots,\bar{K}} q_{ik} + e_i\Big)^2 - \frac{K}{2(1+\gamma)}\Big[\sum_{k=1,\dots,\bar{K}} q_{ik}^2 \\ &+ e_i^2 - \frac{1}{K}\Big(\sum_{k=1,\dots,\bar{K}} q_{ik} + e_i\Big)^2\Big] \end{aligned}$$

where the parameter $\gamma \geq 0$ measures the substitutability of the resources. Resources are perfect substitute when $\gamma \to +\infty$. On the contrary, they are completely differentiate when $\gamma = 0$. This form of utility is used in industrial organization economics to take into account product differentiation in marketing [18] or online advertising [13]. This choice of representation is motivated by the fact that it will enable us to evaluate the impact of the energy mix on the allocation of the suppliers' energy orders and to test the impact of the suppliers' point of view regarding the substitutability of the resources. Indeed, for ideological or marketing reasons, some suppliers might want to promote their virtuous image by certifying the origin of their energy, for instance only green energy with zero carbon emission, or nuclear free, etc. Such criteria are more and more mentioned by marketing advertisers [22]. As a result, it seems quite necessary to introduce in the suppliers' preferences a parameter modeling the resource substitutability.

For a given set of prices $\Big(\tilde{p}_k\Big)_k$, the representative supplier determines for each generator and for the microgrid, the energy orders that maximize his utility:

$$\pi_i = U_i + p_i D_i - \Big(\sum_{k=1,\dots,K} \tilde{p}_k q_{ik} + p_i^e e_i\Big)$$

The quantities of energy bought by supplier s_i to the generators, $(q_{ik})_k$, and to the microgrid M_i, e_i, will be expressed as functions in the prices per unit of production for

[5] The formulation, proposed by Höffler [8], allows for a consistent welfare analysis of the number of available resource categories in the energy market.

the various sources of energy and in the prices per unit of demand response. Furthermore, in Corollary 5, we will prove that the generators' prices are linear functions in the demand reponse prices and in Proposition 7, it will be possible to express demand response prices as functions in the suppliers' prices per demand unit $(p_i)_i$. The prices per demand unit $(p_i)_i$ are inputs of the model. They are designed by the suppliers to maximize their utility but, at the same time, controlled by the Principal, to prevent suppliers' abuse of dominant position and to guarantee that the energy available at the microgrid level will not be wasted.

For any supplier $s_i,\ i = 1, ..., N$, the optimization program takes the form:

$$\begin{aligned} \max_{(q_{ik})_k, e_i} \quad & \pi_i \\ \text{s. t.} \quad & \sum_{j=1,\dots,N} q_{jk} \leq \bar{q}_k, \forall k = 1, ..., K \\ & 0 \leq e_i \leq \bar{q}_i^e \\ & \sum_{k=1,\dots,K} q_{ik} + e_i = D_i \end{aligned} \tag{4}$$

The last constraint traduces the balance between the total quantity of energy which is provided to supplier s_i, on the left, and the quantity of energy which is provided by supplier s_i to meet microgrid M_i's demand, on the right.

2.3 Description of the energy market game

The electrical network energy loss[6] is measured by the sum, over each supplier, of the differences between the total quantity of energy that he bought to the generators and his associated microgrid consumption, defined as the difference between its demand and the quantity that it sells back to the supplier.

Definition 2. *The electrical network energy loss is measured by the following function:*

$$f(\bar{K}, K^s) = \sum_{i=1,\dots,N} \Big[\sum_{k=1,\dots,\bar{K}} q_{ik} - (D_i - e_i) \Big]^2$$

The static game can be broken in two steps: the generators acting first can be seen as the leaders whereas the suppliers acting second, appear to be the followers. Such games are usually called Stackelberg games. They are solved using backward induction [15], [16], [23].

The sequences $(\bar{q}_k)_k$ and $(\bar{q}_i^e)_i$ are determined as realizations of exogenous processes that can be random [17] or take the form of indivual sequences [16]. They represent the productions of the capacities hold by the generators and by the microgrids. They are publicly announced to the Principal.

We now detail the steps of the game:

[6] In Subsections 2.1 and 2.2, the optimal quantities of energy are determined during a peak of consumption. Under the non restrictive assumption that this peak lasts one hour, one energy unit corresponds to one power unit. Therefore, the electrical network loss can be measured in power units, as usual.

(i) $a)$ The generators $(G_k)_k$ determine the number of resources among the K to activate so as to minimize the electrical network energy loss i.e., $\min_{\bar{K}} f(\bar{K}, K^s)$

(i) $b)$ Generator G_k chooses $\tilde{p}_k$ and u_k independently and simultaneously so as to maximize his utility.

(ii) $a)$ The suppliers $(s_i)_i$ determine the number of resources among the $\bar{K}$ to saturate so as to minimize the electrical network energy loss i.e., $\min_{K^s} f(\bar{K}, K^s)$

(ii) $b)$ Supplier s_i chooses $(q_{ik})_k$ and e_i independently and simultaneously so as to maximize his utility.

The side-payments do not intervene in the energy market game.

2.4 Resolution of the energy market game

We let $\tilde{\sigma}_k^* = \Big(\tilde{p}_k^*, u_k^*\Big)$ and $\sigma_i^* = \Big((q_{ik}^*)_k, e_i^*\Big)$ be the pure strategies of generators G_k and suppliers s_i respectively. It is useful to introduce the following notations. The set of all possible strategies available to supplier s_i (resp. generator G_k) will be denoted Σ_i (resp. $\tilde{\Sigma}_k$).

We introduce the variable $\varepsilon > 0$ such that $\varepsilon \to 0$. It will be used throughout the article.

Proposition 3. *The static game admits a unique Nash equilibrium in $(\tilde{\sigma}_k^*)_k, (\sigma_i^*)_i$.*

Proof of Proposition 3. The Lagrangian function resulting from supplier s_i's constrained optimization problem as described by the system of Equations (4) and evaluated at the optimum $\Big((q_{ik})_k, e_i\Big)_i$, takes the form:

$$L_i = \pi_i + \sum_{k=1,\dots,K^s} \mu_k \Big(\sum_{j=1,\dots,N} q_{jk} - \bar{q}_k \Big) + \lambda_i \Big[\sum_{k=1,\dots,\bar{K}} q_{ik} + e_i - D_i \Big]$$

where the multipliers μ_k, λ_i are real numbers. The coefficient μ_k is identical for all the suppliers because the associated inequality constraint is the same for all of them. According to the Karush-Kühn-Tucker conditions, we have the following relations for the inequality constraints:

$$\mu_k \Big(\sum_{j=1,\dots,N} q_{jk} - \bar{q}_k \Big) = 0$$

and $\mu_k \geq 0, \forall k = K^s + 1, ..., \bar{K}$. But for $k = K^s + 1, ..., \bar{K}$, resource k is never saturated i.e., $\sum_{j=1,\dots,N} q_{jk} - \bar{q}_k < 0$. This implies in turn that $\mu_k = 0, \forall k = K^s + 1, ..., \bar{K}$. Therefore, it is useless to introduce the inequality constraints in the Lagrangian function, at the optimum.

To optimize supplier s_i decisions in the quantity of energy to buy to the generators and to microgrid M_i, we start by differentiating the Lagrangian function L_i with respect to q_{ik} and to e_i. The differentiation of the Lagangian function L_i with respect to q_{ik} takes two forms depending on the position of k with respect to K^s:

- If $k \in \{1, ..., K^s\}$ then $\frac{\partial L_i}{\partial q_{ik}} = \frac{\partial \pi_i}{\partial q_{ik}} + \mu_k + \lambda_i$.
- If $k \in \{K^s + 1, ..., \bar{K}\}$ then $\frac{\partial L_i}{\partial q_{ik}} = \frac{\partial \pi_i}{\partial q_{ik}} + \lambda_i$.

While the differentiation of L_i with respect to e_i gives: $\frac{\partial L_i}{\partial e_i} = \frac{\partial \pi_i}{\partial e_i} + \lambda_i$.

At the optimum in $\Big((q_{ik})_k, e_i\Big)_i$, we have the relation: $\frac{\partial L_i}{\partial q_{ik}} = \frac{\partial L_i}{\partial e_i}$. Depending on the position of k with respect to K^s, two cases should be distinguished:

- If $k \in \{K^s + 1, ..., \bar{K}\}$ then we obtain:

$$q_{ik} = e_i + \frac{1+\gamma}{K}(p_i^e - \tilde{p}_k), \ \forall k = K^s + 1, ..., \bar{K} \tag{5}$$

- If $k \in \{1, ..., K^s\}$ then we obtain:

$$q_{ik} = e_i + \frac{1+\gamma}{K}(p_i^e - \tilde{p}_k) + \frac{1+\gamma}{K}\mu_k, \ \forall k = 1, ..., K^s \tag{6}$$

By summation of Equation (6) over $i = 1, ..., N$, we get:

$$\mu_k = \frac{K}{N(1+\gamma)}\Big[\bar{q}_k - \sum_{i=1,\ldots,N} e_i\Big] - \frac{1}{N}\sum_{i=1,\ldots,N}(p_i^e - \tilde{p}_k) \tag{7}$$

By substitution of Equation (5) in the last constraint of the system of Equations (4), we infer the relation:

$$\sum_{k=1,\ldots,K^s} q_{ik} + (\bar{K} - K^s + 1)e_i = D_i - \frac{1+\gamma}{K}\Big[(\bar{K} - K^s)p_i^e - \sum_{k=K^s+1,\ldots,\bar{K}} \tilde{p}_k\Big] \tag{8}$$

Then, by substitution of Equation (6) in Equation (8), we obtain the optimal quantity for supplier s_i to buy to microgrid M_i as a function of the difference between the price fixed by microgrid M_i and the generators' prices and of the Lagrange multipliers for all the saturated generators:

$$e_i = \frac{1}{\bar{K}+1}D_i - \frac{1+\gamma}{K(\bar{K}+1)}\sum_{k=1,\ldots,\bar{K}}(p_i^e - \tilde{p}_k) - \frac{1+\gamma}{K(\bar{K}+1)}\sum_{k=1,\ldots,K^s}\mu_k \tag{9}$$

For the sake of simplicity, we let: $\delta_{ik} = p_i^e - \tilde{p}_k$ be the difference between the price fixed by microgrid M_i and generator G_k's price. Substituting μ_k, as described in Equation (7), in e_i expression as obtained in Equation (9), we infer the following relation between the quantities of energy bought by the suppliers to the microgrids and the price differences:

$$(1 - \frac{K^s}{\bar{K}+1})e_i - \frac{K^s}{\bar{K}+1}\sum_{j\neq i} e_j = \frac{1}{\bar{K}+1}D_i - \frac{1+\gamma}{K(\bar{K}+1)}\sum_{k=1,\ldots,\bar{K}}\delta_{ik} - \frac{1}{\bar{K}+1}\sum_{k=1,\ldots,K^s}\bar{q}_k + \frac{1+\gamma}{NK(\bar{K}+1)}\sum_{j=1,\ldots,N}\sum_{k=1,\ldots,K^s}\delta_{jk} \tag{10}$$

We make the following assumptions:

- Without loss of generalities, we assume that it is the microgrid M_1 which generates the highest quantity of energy i.e., $e_1 = \max_{j=1,...,N}\{e_j\}$.
- The demand responses performed by the other microgrids can be expressed as linear functions of the demand response performed by microgrid M_1. This means that there exists a real sequence $\{\beta(j)\}_{j=2,...,N}$ such that $0 \leq \beta(j) \leq 1$ and $e_j = \beta(j)e_1, \forall j = 2, ..., N$.

According to the above assumptions, Equation (10) can then be re-written. We obtain:

$$
\begin{aligned}
e_1 &= \Big(\frac{\bar{K}+1}{(\bar{K}+1) - \Big[1 + \sum_{j=2,\dots,N} \beta(j)\Big] K^s}\Big) \Big[\frac{1}{\bar{K}+1}(D_1 - \sum_{k=1,\dots,K^s} \bar{q}_k) \\
&- \nu(\sum_{k=1,\dots,\bar{K}} \delta_{1k} - \frac{1}{N} \sum_{j=1,\dots,N} \sum_{k=1,\dots,K^s} \delta_{jk})\Big] \qquad (11)
\end{aligned}
$$

where we set: $\nu \equiv \nu(K, \gamma) = \frac{1+\gamma}{K(K+1)}$.

To optimize generator G_k's optimal decision in price and effort, two cases should be distinguished depending on the position of k with respect to K^s.

- If $k \in \{K^s+1, ..., \bar{K}\}$ then resource k is not saturated i.e., $\sum_{j=1,\dots,N} q_{jk} < \bar{q}_k$. For $j = 1, ..., N$, any q_{jk} can be expressed as a linear function in e_1. We then start by substituting e_1 expression, as defined in Equation (11), in the $(q_{jk})_j$, derived from Equation (5). $(q_{jk})_j$ are components of generator G_k's utility, which is defined in Equation (2). By differentiation with respect to $\tilde{p}_k$, we obtain:

$$
\tilde{p}_k = \frac{\sum_{j=1,\dots,N} \tilde{p}^a(j,k) + \frac{1+\gamma}{K} \sum_{j=1,\dots,N} p^e_j}{2\Big[\frac{N\nu(\bar{K}+1)}{(\bar{K}+1) - [1 + \sum_{j=2,\dots,N} \beta(j)] K^s} - N\frac{1+\gamma}{K}\Big]} - \frac{1}{2}c_k \qquad (12)
$$

where we set:

$$
\begin{aligned}
\tilde{p}^a(j,k) &= \frac{\bar{K}+1}{(\bar{K}+1) - [1 + \sum_{m=2,\dots,N} \beta(m)] K^s} \Big[\frac{1}{\bar{K}+1}(D_j - \sum_{l=1,\dots,K^s} \bar{q}_l) \\
&- \nu(\sum_{l=1,\dots,\bar{K}, l\neq k} \delta_{jl} + p^e_j - \frac{1}{N} \sum_{m=1,\dots,N} \sum_{l=1,\dots,K^s} \delta_{ml})\Big]
\end{aligned}
$$

Additionally, we introduce the auxiliary variable:

$$
c^a = \frac{K\Big((\bar{K}+1) - [1 + \sum_{j=2,\dots,N} \beta(j)] K^s\Big)}{N(1+\gamma)\Big((\bar{K}+1) - [1 + \sum_{j=2,\dots,N} \beta(j)] K^s\Big) - NK\nu(\bar{K}+1)}
$$

Differentiating generator G_k's utility, as defined in Equation (2), with respect to c_k and by substitution of $\tilde{p}_k$, we obtain:

$$c_k = \frac{5}{3}c^a \sum_{j=1,\dots,N} \Big(\tilde{p}^a(j,k) + \frac{1+\gamma}{\bar{K}} p_j^e\Big) \tag{13}$$

But, according to Equation (3), we have: $\frac{c_k}{c_{k0}} = \exp(-\rho\psi(u_k))$. Function $\psi(.)$ being continuous and strictly increasing by assumption, it is invertible. As a result, it is possible to infer generator G_k's optimal effort as a function of his optimal cost as described just above: $u_k = \psi^{-1}(-\frac{1}{\rho}\log\frac{c_k}{c_{k0}})$.

- If $k \in \{1, ..., K^s\}$ then resource k is saturated i.e., $\sum_{j=1,\dots,N} q_{jk} = \bar{q}_k$. Using the previous assumption that $e_j = \beta(j)e_1, \forall j = 2, ..., N$, we substitute e_1, as obtained in Equation (11), in $(q_{jk})_j$, as defined in Equation (6). We infer that $(q_{jk})_j$ does not depend on $\tilde{p}_k$ since e_1 does not depend anymore on $\tilde{p}_k$. This means that generator G_k's utility, as defined in Equation (2), is linear increasing in $\tilde{p}_k$. A solution is to choose: $\tilde{p}_k = \tilde{p}_{K^s+1} - \Big((K^s+1) - k\Big)\varepsilon$ with $\varepsilon > 0$ and $\varepsilon \to 0$.

  Differentiating $\tilde{\pi}_k$ with respect to u_k we obtain:

$$\frac{\partial \tilde{\pi}_k}{\partial u_k} = \rho c_{k0}\bar{q}_k \psi'(u_k)\exp\Big(-\rho\psi(u_k)\Big) - 1$$

  Generator G_k's optimal effort belongs to the following set:

$$u_k = \Big\{u \geq 0 | \psi'(u)\exp\Big(-\rho\psi(u)\Big) = \frac{1}{\rho c_{k0}\bar{q}_k}\Big\}$$

  The solution is unique since the function $\psi'(u)\exp\Big(-\rho\psi(u)\Big)$ is strictly decreasing in u. Indeed by differentiation with respect to u, we obtain:

$$\underbrace{\psi''(u)}_{<0}\underbrace{\exp\Big(-\rho\psi(u)\Big)}_{>0} + \underbrace{\psi'(u)}_{>0}\underbrace{\Big(-\rho\psi'(u)\Big)}_{<0}\underbrace{\exp\Big(-\rho\psi(u)\Big)}_{>0} < 0$$

$\square$

Corollary 4. *If supplier s_1 buys microgrid M_1's total production, the microgrid's capacity can be expressed as a linear function in the distances between, on the one hand, its demand and the sum of the saturated generators' capacities and, on the other hand, between the $\sum_{k=1,\dots,\bar{K}} \delta_{1k}$ and the mean of the $\Big(\sum_{k=1,\dots,K^s}\delta_{jk}\Big)_{j=1,\dots,N}$:*

$$\begin{aligned}\bar{q}_1^e &= \frac{1}{\bar{K}+1-[1+\sum_{j=2,\dots,N}\beta(j)]K^s}\Big\{(D_1 - \sum_{k=1,\dots,K^s}\bar{q}_k) \\ &- \nu(\bar{K}+1)\Big(\sum_{k=1,\dots,\bar{K}}\delta_{1k} - \frac{1}{N}\sum_{j=1,\dots,N}\sum_{k=1,\dots,K^s}\delta_{jk}\Big)\Big\}\end{aligned}$$

Proof of Corollary 4. If supplier s_1 buys the total quantity of energy proposed by microgrid M_1 then $e_1 = \bar{q}_1^e$. By substitution of this equality in Equation (11), we derive that microgrid M_1's capacity depends linearly on the differences between its demand and the saturated generators' capacities $(D_1 - \sum_{k=1,...,K^s} \bar{q}_k)$. Furthermore, microgrid M_1's capacity depends linearly on the coefficiented differences between its demand response price and the other producers' prices ; this factorization coefficient can be expressed as: $\sum_{k=1,...,\bar{K}} \delta_{1k} - \frac{1}{N} \sum_{j=1,...,N} \sum_{k=1,...,K^s} \delta_{jk} = \bar{K}\Big((1 - \frac{K^s}{\bar{K}})p_1^e - \frac{K^s}{\bar{K}} \sum_{j=2,...,N} p_j^e - \frac{1}{\bar{K}} \sum_{k=K^s+1,...,\bar{K}} \tilde{p}_k\Big)$. Among the active generators, the saturated generators' impact is already evaluated in the first part of the equation.

□

Corollary 5. *For any generator G_k with $k = K^s + 1, ..., \bar{K}$, his price per unit of energy can be expressed as a linear function in the sum of the suppliers' prices per unit of demand response:*

$$\tilde{p}_k = \eta \sum_{j=1,...,N} p_j^e + \zeta_k$$

where:

$$\eta = \frac{4c^a}{\eta_d}\Big(\frac{1+\gamma}{K} - \frac{\nu(\bar{K}+1)}{(\bar{K}+1) - [1 + \sum_{j=2,...,N} \beta(j)]K^s}\Big)$$

$$\begin{aligned}
\zeta_k &= \frac{2c^a \nu N(\bar{K}+1)}{\eta_d + 2c^a \nu N(\bar{K}+1)(\bar{K} - K^s + 1)}\Big[- \bar{K}(\bar{K}+1) + (K^s+2)(K^s+3) \\
&- \frac{2}{\nu N(\bar{K}+1)}\Big(\sum_{j=1,...,N} \frac{D_j - \sum_{k=1,...,K^s} \bar{q}_k}{\bar{K} + 1 - [1 + \sum_{l=2,...,N} \beta(l)]K^s}\Big) + \frac{N(1+\gamma)}{K} + 2\varepsilon\Big] \\
&+ (k - K^s - 1)\varepsilon
\end{aligned}$$

and $\eta_d = 3(\bar{K}+1) - 3[1 + \sum_{j=2,...,N} \beta(j)]K^s + 2c^a \nu N(\bar{K}+1)(\bar{K} - K^s + 1)$.

Proof of Corollary 5. Going back to $\tilde{p}_{K^s+1}$ expression at equilibrium given in Equation (12) and substituting the value of c_k as derived in Equation (13), we obtain a simplified expression for the generator G_{K^s+1}'s price per demand unit: $\tilde{p}_{K^s+1} = -\frac{4}{3}c^a \sum_{j=1,...,N} \Big(\tilde{p}^a(j,k) + \frac{1+\gamma}{K} p_j^e\Big)$. By substitution of the analytical expression of

$\tilde{p}^a(j,k)$ in the previous equation, we obtain the following relation:

$$\begin{aligned}
& \tilde{p}_{K^s+1} + \frac{4}{3}c^a \frac{\nu N(\bar{K}+1)}{(\bar{K}+1) - [1+\sum_{j=2,...,N}\beta(j)]K^s} \sum_{l=K^s+2,...,\bar{K}} \tilde{p}_l \\
= & -\frac{4}{3}c^a \sum_{l=1,...,N} [\frac{1}{(\bar{K}+1) - [1+\sum_{j=2,...,N}\beta(j)]K^s}(D_l - \sum_{k=1,...,K^s}\bar{q}_k) \\
+ & (\frac{1+\gamma}{K} - \frac{\nu(\bar{K}+1)}{(\bar{K}+1) - [1+\sum_{j=2,...,N}\beta(j)]K^s})p_l^e]
\end{aligned} \tag{14}$$

According to this equation, the generators' price per demand unit are linearly dependent. We make the assumption that:

$$\tilde{p}_k = \tilde{p}_{K^s+1} + \Big(k - (K^s+1)\Big)\varepsilon, \forall k = K^s+1, ..., \bar{K} \tag{15}$$

Substituting Equation (15) in Equation (14), we infer that it is possible to express $\tilde{p}_{K^s+1}$ as a function of $(p_l^e)_{l=1,...,N}$ and of the game parameters exclusively. We obtain:

$$\begin{aligned}
\tilde{p}_{K^s+1} = & \frac{\frac{2}{3}c^a\nu N(\bar{K}+1)}{\bar{K}+1-\Big[1+\sum_{j=2,...,N}\beta(j)\Big]K^s + \frac{4}{3}(\bar{K}-K^s+1)c^a\nu N(\bar{K}+1)} \\
& \Big\{(K^s+2)(K^s+3) + 2\varepsilon - \bar{K}(\bar{K}+1) \\
- & \frac{2}{\nu(\bar{K}+1)}\Big[\frac{1}{\bar{K}+1-[1+\sum_{j=2,...,N}\beta(j)]K^s}(\sum_{l=1,...,N}D_l - \sum_{k=1,...,K^s}\bar{q}_k) \\
+ & \frac{1}{N}(\frac{1+\gamma}{K} - \frac{\nu(\bar{K}+1)}{\bar{K}+1-[1+\sum_{j=2,...,N}\beta(j)]K^s})\sum_{l=1,...,N}p_l^e\Big]\Big\}
\end{aligned}$$

By substitution in Equation (15) and by identification, we infer the analytical expressions of η, η_d and ζ_k. □

We recall that for $k = 1, ..., K^s$, the expression of the generators $(G_k)_{k=1,...,K^s}$ prices at equilibrium was obtained in Proposition 3 proof:

$$\tilde{p}_k = \tilde{p}_{K^s+1} - \Big((K^s+1) - k\Big)\varepsilon$$

2.5 Description of the Nash equilibrium computation algorithm

Proposition 6. *There exist only K admissible combinations of $(a_k)_k$ satisfying the price constraints defined in Equation (1).*

Proof of Proposition 6. According to the ordering assumption made on the $\tilde{p}_k$ in Equation (1), we necessarily have $a_1 = 1$ since the microgrids' total demand cannot be zero. Furthermore, if $a_l = 0$, necessarily $a_{l+1} = 0$. This implication holds for any $l = 2, ..., K-1$. As a result, the set of admissible combinations of active generators contains all the combinations where the $l = 1, ..., K-1$ first generators are active while the others are closed and the combination where all the generators are active. This leads to a total of K admissible combinations for $(a_k)_k$. □

As already mentioned in Subsection 2.3, the generators' and the microgrids' capacities $(\bar{q}_k)_k$ and $(\bar{q}^e_i)_i$ are supposed to be exogenous to the model. The microgrids' demands $(D_i)_i$ are fixed at their peak value plus a reservation margin. In addition to the demand unit prices $(p_i)_i$ which are designed by the suppliers but controlled by the Principal to avoid abuse of dominant positions and energy wastings, we recall below the list of the game parameters that will need to be calibrated:

- N, K the market structure
- c_{k0} generator G_k's initial marginal cost
- $\gamma \geq 0$ the energy substitutability parameter
- $\rho > 0$ the technological opportunity
- $\varepsilon > 0$ such that $\varepsilon \to 0$
- $\{\beta(j)\}_{j=2,...,N}$ such that $\beta(j) \in [0; 1]$ and $e_j = \beta(j)e_1, \forall j = 2, ..., N$

We describe below the algorithm which enables the computation of the game Nash equilibrium.

Algorithm 1 **Nash Equilibrium Computation**

1: For each of the K admissible combinations of $(a_k)_k$ determine $K_s \in [\![0; \sum_{k=1,...,K} a_k]\!]$ minimizing $f(\sum_{k=1,...,K} a_k, K_s)$.

2: Evaluate numerically the Nash equilibrium $(\sigma_k)_k, (\sigma_i)_i$

3: Keep in memory the optimum $K_s^*\Big((a_k)_k\Big)$ and the value of the objective function $f\Big(\sum_{k=1,...,K} a_k, K_s^*((a_k)_k)\Big)$

4: The generators $(G_k)_k$ determine the combination $(a_k)_k^*$ minimizing the set of the admissible values for the objective function $f\Big(\sum_{k=1,...,K} a_k, K_s^*((a_k)_k)\Big)$ over the set of admissible combinations $(a_k)_k$

The number of active generators are determined as an output of the algorithm i.e., $\bar{K} = \sum_{k=1,...,K} (a_k)^*$.

2.6 Analytical computation of the demand response price

The price proposed by the supplier s_1 per unit of demand response can be optimized by the Principal to guarantee that the microgrid's welfare remains non-negative and to avoid the supplier's abuse of dominant position. We make the assumption that there exists a positive real sequence $\{\theta(j)\}_{j=2,...,N}$ such that $p_j^e = \theta(j)p_1^e, \forall j = 2, ..., N$. Furthermore, since microgrid M_1 generates the higher demand response, it is realistic to assume that this high level, which requires a real effort in terms of flexibility and storage capacity at the microgrid's level, is consecutive to a high demand at the supplier's level. Therefore, we set: $e_1 = \bar{q}_1^e$.

We go back to the expression of e_1, as derived in Corollary 4, where we expressed $\bar{q}_1^e$ as a function of $(p_j^e)_{j=1,...,N}$ and of $(\tilde{p}_k)_{k=K^s+1,...,\bar{K}}$. Substituting the analytical expression of $\tilde{p}_k$ for $k = K^s + 1, ..., \bar{K}$, as obtained in Corollary 5, we derive the following relation:

$$\begin{aligned}
\bar{q}_1^e &= \frac{1}{\bar{K}+1-[1-\sum_{j=2,...,N}\beta(j)]K^s}\Big(D_1 - \sum_{k=1,...,K^s}\bar{q}_k \\
&+ \nu(\bar{K}+1)\sum_{k=K^s+1,...,\bar{K}}\zeta_k\Big) + \frac{\nu(\bar{K}+1)}{\bar{K}+1-[1-\sum_{j=2,...,N}\beta(j)]K^s} \\
&\Big[\Big((\bar{K}-K^s)\eta + \frac{K^s}{N}\Big)\Big(1+\sum_{j=2,...,N}\theta(j)\Big) - \bar{K}\Big]p_1^e
\end{aligned}$$

Therefore, $\bar{q}_1^e$ is a linear function in p_1^e. We set: $\bar{q}_1^e = w_1 + v_1 p_1^e$ where, by identification:

$$\begin{aligned}
w_1 &= \frac{1}{\bar{K}+1-[1-\sum_{j=2,...,N}\beta(j)]K^s}\Big(D_1 - \sum_{k=1,...,K^s}\bar{q}_k + \\
&+ \nu(\bar{K}+1)\sum_{k=K^s+1,...,\bar{K}}\zeta_k\Big) + \frac{\nu(\bar{K}+1)}{\bar{K}+1-[1-\sum_{j=2,...,N}\beta(j)]K^s} \\
v_1 &= \Big((\bar{K}-K^s)\eta + \frac{K^s}{N}\Big)\Big(1+\sum_{j=2,...,N}\theta(j)\Big) - \bar{K}
\end{aligned}$$

Proposition 7.

- *If $v_1 \geq 0$ then microgrid M_1's social welfare is non-negative if, and only if, $p_1^e \geq \frac{-w_1+\sqrt{w_1^2+4v_1p_1D_1}}{2v_1}$.*
- *If $\frac{-w_1^2}{4p_1D_1} \leq v_1 < 0$ and $w_1 \geq 0$ then microgrid M_1's social welfare is non-negative if, and only if, $\frac{-w_1+\sqrt{w_1^2+4v_1p_1D_1}}{2v_1} \leq p_1^e \leq \frac{-w_1-\sqrt{w_1^2+4v_1p_1D_1}}{2v_1}$.*
- *Otherwise, microgrid M_1's social welfare always remains negative.*

Proof of Proposition 7. Microgrid M_1's social welfare, sw_1, is defined as the difference between the revenue received from supplier s_1 and the revenue paid to supplier s_1 i.e., $sw_1 = p_1^e e_1 - p_1 D_1$. The microgrid's net utility resulting from the benefit perceived through the consumption of D_1 unit of demand is set equal to zero[7]. Using the fact that $\bar{q}_1^e$ can be rewritten as: $\bar{q}_1^e = w_1 + v_1 p_1^e$, microgrid M_1's social welfare can be rewritten to give:

$$\begin{aligned} sw_1 &= p_1^e e_1 - p_1 D_1 \\ &= (p_1^e)^2 v_1 + p_1^e w_1 - p_1 D_1 \end{aligned}$$

Microgrid M_1's social welfare can be interpreted as a second order polynomial equation in p_1^e. Depending on the sign of v_1, two cases should be distinguished.

If $v_1 \geq 0$ then sw_1 admits 2 roots in p_1^e of opposite sign since the constant term of the polynomial equation is negative. The demand response price being positive, we infer that in case where $v_1 \geq 0$, $sw_1 \geq 0$ if, and only if, p_1^e is greater than the highest value root.

If $v_1 < 0$ then provided the polynomial equation discriminant $w_1^2 + 4v_1 p_1 D_1 \geq 0$ i.e., $v_1 \geq -\frac{w_1^2}{4p_1 D_1}$, microgrid M_1's social welfare is non-negative if, and only if, p_1^e belongs to the interval which bounds are defined by the two roots of the polynomial equation. These two roots share the same sign since the constant term of the polynomial equation is negative. There are both non-negative if, and only if, the maximum of the polynomial equation is reached in a positive demand response price. But, the polynomial equation reaches its maximum in $p_1^e = \frac{-w_1}{2v_1}$. Therefore, it is positive if, and only if, $w_1 \geq 0$. $\square$

Since it is the price fixed by supplier s_1 for each unit of demand response, he will choose the smallest value avoiding him reaching an abusive dominant position. We conclude that:

- If $v_1 \geq 0$ then $p_1^e = \frac{-w_1 + \sqrt{w_1^2 + 4v_1 p_1 D_1}}{2v_1} + \varepsilon$.
- If $\frac{-w_1^2}{4p_1 D_1} \leq v_1 < 0$ and $w_1 \geq 0$ then $p_1^e = \frac{-w_1 + \sqrt{w_1^2 + 4v_1 p_1 D_1}}{2v_1} + \varepsilon$.
- The case $v_1 < \frac{-w_1^2}{4p_1 D_1}$ or $w_1 < 0$ corresponds either to an abuse of dominant position of the supplier over the microgrid, since this latter's social welfare always remains negative in this case, or to the case where the generators' capacities of production are so large that the suppliers do not need to buy very much energy to the microgrids. Therefore, the Principal will control that the game parameters and the demand prices $(p_i)_i$ are calibrated so as to avoid falling in this latter case.

These results are consistent with the demand response operators' intense requirements to remunerate voluntary demand shiftings by indexing its price on the demand price[8].

[7] Generalizations to any other constant value are trival.

[8] At the moment, residential consumers are not remunerated for voluntary demand shiftings. The experienced benefit results exclusively from the potential decrease of the electricity bill due to the drop of the demand. However, the "report" effect is possible which might prevent monetary savings.

In the model described in this article, the agents' strategy at equilibrium depends on their private information about the capacities $(\bar{q}_k)_k$ and $(\bar{q}_i^e)_i$. Furthermore, the generators' utilities depend on the side-payments, $(\mathcal{T}_k)_k$, delivered by the Principal to the generators, as captured in Equation (2). These side-payments represent a first lever of action, with which the Principal can play while designing the capacity market rules. The other lever of action appears through the delivery of capacity certificates. Both levers will be used as powerful tools for the design of the capacity market rules.

3 Mechanism design for the capacity market

The game studied in this article is with asymmetric information since the true capacities $(\bar{q}_k^0)_k$ and $(\bar{q}_i^{e0})_i$ are known only by their producer. These constitute the private information, also known as the types [23], [24], of the generators and of the microgrids. At the network level, the generators and the microgrids monitor their productions using sensors deployed on their park of production or in the houses of the eco-neighborhoods, for solar pannels and storage devices [17]. Each agent (generators and microgrids) reports his estimated capacity production to the Principal: $\hat{q}_k$ for generator G_k and $\hat{q}_i^e$ for microgrid M_i. The reported capacities are then publicly announced on the capacity market. The major problem is that the reports can be biased i.e., there is no reason for the agents to report their true type a priori! In the game theory literature, the tendancy of the agents to distort the information by reporting the risk and the monetary consequences on the other agents is known as moral hazard [23]. Here, moral hazard might appear if the generators perform capacity retention or, on the contrary, declare more capacity than they have [5], [6], [11], [12]. The first situation goal is to create scarcity and then, to make the capacity price increase artificially. In the second situation, the generators invest less in the capacities than what they declared. In both cases, the suppliers and the whole market operations are altered.

The mechanism design problem is to implement an optimal system wide solution to a decentralized optimization problem with self-interested agents with private information [24]. In other words, the mechanism design problem is to implement rules of the game such as defining possible strategies and the method used to select an outcome based on agents' strategies, to implement the solution to the social choice function despite agents' self-interest. Here, the agents have quasi-linear utility functions [24] with types $(\bar{q}_k^0)_k$ and $(\bar{q}_i^{e0})_i$ containing their private information about the capacities. Quasi-linear utilities make it straightforward to transfer utility across agents, via side-payments which can take the form of punishments.

We now describe the social choice function. The Principal's role is to master the whole system behavior but he does not take part to the definition of the energy orders and prices ; neither between the microgrids and the suppliers, nor between the suppliers and the generators. However, he is concerned with the system wide balance. As a result, he needs to evaluate a posteriori:

$$\phi = \sum_{k=1,\ldots,\bar{K}} \bar{q}_k^0 - \sum_{i=1,\ldots,N} \left(D_i - \bar{q}_i^{e0} \right)$$

$\phi \equiv \phi\Big((\bar{q}_k^0)_k, (\bar{q}_i^{e0})_i\Big)$ measures the difference between the (true) total capacity production and the total consumption of the microgrids. The case $\phi > 0$ will be avoided by the constraints imposed in the certification process that will be explained later. If $\phi = 0$, the system is perfectly balanced since capacity equals demand. If $\phi < 0$, capacity is missing. The Principal will need to invest in order to construct the missing capacity. As already mentioned in the Introduction, this mechanism is known as feedback mechanism in the capacity market literature [26].

Definition 8. *A mechanism* $m = \Big(\prod_i \Sigma_i \prod_k \tilde{\Sigma}_k, g(.)\Big)$ *defines the set of strategies available to each agent, and an outcome rule* $g : \prod_i \Sigma_i \prod_k \tilde{\Sigma}_k \to \mathcal{O}$ *such that* $g\Big((\sigma_i)_i, (\tilde{\sigma}_k)_k\Big)$ *is the outcome implemented by the mechanism.*

Given mechanism m with outcome function $g(.)$ we say that a mechanism implements social choice function $\phi\Big((\bar{q}_i^{e0})_i, (\bar{q}_k^0)_k\Big)$ if the outcome computed with equilibrium agent strategies is a solution to the social choice function for all possible agent information. More explicitely, mechanism m implements social choice function $\phi\Big((\bar{q}_i^{e0})_i, (\bar{q}_k^0)_k\Big)$ if $g\Big((\sigma_i^*(\bar{q}_i^{e0}))_i, (\tilde{\sigma}_k^*(\bar{q}_k^0))_k\Big) = \phi\Big((\bar{q}_i^{e0})_i, (\bar{q}_k^0)_k\Big)$ for all $(\bar{q}_i^{e0})_i, (\bar{q}_k^0)_k \in \mathbb{R}_+^{K+N}$ where $(\sigma_i^*)_i, (\tilde{\sigma}_k^*)_k$ is the equilibrium solution of the game induced by m. Its analytical expression is detailed in Propositions 3 and 7 proofs. The outcome rules, defined by function $g(.)$, contain the side-payments transferred by the Principal to the generators and the quantities of certified capacities for the generators and for the microgrids.

The mechanism m, which enables the definition of rules over the capacity market, should satisfy the three properties below:

- System wide balance (SWB) i.e., $\phi = 0$.
- Individual rationality (IR) i.e., the generators can always achieve as much utility from entering the capacity market as without participation. The refusal to participate to the market leads to a zero utility for the generators[9]. The IR property guarantees the market opening to new potential investors.
- Incentive compatibility (IC) i.e., the generators report truthful information about their production in equilibrium. The IC property avoids moral hazard to occur.

In the following, we will compute the analytical expression of the Principal's effort and detail the hierarchical optimization problem, guaranteeing the satisfaction of the SWB property.

To force the generators and the microgrids to report their true type, the Principal will certify the capacities maximizing their coefficiented social welfare:

$$\sum_{k=1,\dots,\bar{K}} \tilde{\alpha}_k \tilde{\pi}_k + \sum_{i=1,\dots,N} \alpha_i \pi_i$$

[9]In Section 2, we chose to neglect the energy exchanges between the suppliers. On the contrary, exports and imports are possible between individual generators producing the same category of energy. In our model, the generators being aggregated by resource categories, exchanges occur among individual generators belonging to the same category but not between categories.

Introducing coefficients $(\tilde{\alpha}_k)_k, (\alpha_i)_i$ is equivalent with defining a priority profile for the generators and for the microgrids. Its definition depends on the country's potential regarding their geological richness (natural richness in oil, coal or gas, etc.) as well as on geographic (maritime or river conditions favoring the establishment of off-shore wind farms or dams, etc.) and climatic aspects (strong sunlight and wind, etc.), and finally, on political considerations. As a result, it can be thought as the country energy mix. We make the assumption that the coefficients $(\tilde{\alpha}_k)_k$ are ordered so that $\tilde{\alpha}_k \geq \tilde{\alpha}_{k+1}$ i.e., coefficient $\tilde{\alpha}_k$ is associated with the generator having the k-th highest priority level. The microgrids will only be called in case where the total capacity produced by the generators is not sufficient to meet the microgrids' demand. This implies that $\tilde{\alpha}_{\bar{K}} \geq \alpha_1$. The priority profile for the microgrids follows the index order. In other words, microgrid M_i has priority over microgrid M_j provided $i < j$.. Besides, it is realistic, regarding the assumptions made in Subsection 2.4, to assume that it is microgrid M_1 which produces the highest demand response. Finally, the priority coefficients are defined so that $\tilde{\alpha}_k, \alpha_i \geq 0, \forall k = 1, ..., \bar{K}, \forall i = 1, ..., N$ and $\sum_{k=1,\ldots,\bar{K}} \tilde{\alpha}_k + \sum_{i=1,\ldots,N} \alpha_i = 1$. They are hidden to the agents and known only by the Principal who masters the capacity market. $0 \leq \bar{q}_k^* \leq \hat{q}_k$ is the quantity of energy produced by generator G_k and certified by the Principal. $0 \leq \bar{q}_i^{e*} \leq \hat{q}_i^e$ is the quantity of energy delivered by microgrid M_i and certified by the Principal. For the mechanism m to check SWB, it is necessary to have:

$$\begin{aligned} \left(\bar{q}_k^*\right)_k &= \arg\max_{(\bar{q}_k)_k} \sum_{k=1,\ldots,\bar{K}} \tilde{\alpha}_k \tilde{\pi}_k \\ &\quad \text{s. t.} \sum_{k=1,\ldots,\bar{K}} \bar{q}_k \leq \sum_{i=1,\ldots,N} D_i \\ &\qquad 0 \leq \bar{q}_k \leq \hat{q}_k, \forall k = 1, ..., \bar{K} \end{aligned} \tag{16}$$

and

$$\begin{aligned} (\bar{q}_i^{e*})_i &= \arg\max_{(\bar{q}_i^e)_i} \sum_{i=1,\ldots,N} \alpha_i \pi_i \\ &\quad \text{s. t.} \sum_{i=1,\ldots,N} \bar{q}_i{}^e \leq \sum_{i=1,\ldots,N} D_i - \sum_{k=1,\ldots,\bar{K}} \bar{q}_k^* \\ &\qquad 0 \leq \bar{q}_i^e \leq \hat{q}_i^e, \forall i = 1, ..., N \end{aligned} \tag{17}$$

The optimization problem described by Equations (16) and (17) is a hierarchical one. This means that it can be broken into $\bar{K} + N$ sub-problems of optimization. For the generator having the k-th highest priority level, the capacities and the reports associated to the ordered sequence of generators are denoted $\bar{q}(k), \hat{q}(k)$ and the utility, $\tilde{\pi}(k)$.

The k-th optimization problem is:

$$\begin{aligned}
\bar{q}(k)^* &= \arg\max_{\bar{q}(k)} \tilde{\pi}(k)|_{\left(\bar{q}(l)^*\right)_{l=1,\ldots,k-1}} \\
&\quad \text{s. t. } 0 \leq \bar{q}(k) \leq \sum_{i=1,\ldots,N} D_i - \sum_{l=1,\ldots,k-1} \bar{q}(l)^* \\
&\quad\quad 0 \leq \bar{q}(k) \leq \hat{q}(k)
\end{aligned}$$

and the $(k+i)$-th optimization problem is:

$$\begin{aligned}
\bar{q}_i^{e*} &= \arg\max_{\bar{q}_i^e} \pi_i|_{(\bar{q}_l^*)_{l=1,\ldots,\bar{K}},(q_j^{e*})_{j=1,\ldots,i-1}} \\
&\quad \text{s. t. } 0 \leq \bar{q}_i^e \leq \sum_{i=1,\ldots,N} D_i - \sum_{l=1,\ldots,\bar{K}} \bar{q}_l^* - \sum_{j=1,\ldots,i-1} \bar{q}_i^{e*} \\
&\quad\quad 0 \leq \bar{q}_i^e \leq \hat{q}_i^e
\end{aligned}$$

We let $u_P \in \mathbb{R}_+^{\bar{K}}$ be the vector of efforts made by the Principal. In particular, $u_P(k) \geq 0$ is the effort made by the Principal toward generator G_k. This effort characterizes the side-payment between the Principal and the generator. There is no side-payment between the Principal and the microgrids because the microgrids are already paid by the suppliers for their demand response ; these payments are supposed to compensate the effort made by the eco-neighborhood inhabitants in storage equipment, in the monitoring of their online consumption through boxes, sensors and in related energy management services. According to the constraints imposed in Equations (16) and (17) on the capacity to certify, the function ϕ is always negative or null i.e., $\phi \leq 0$. We consider a mechanism m where the side-payments are defined as follows: $\mathcal{T}_k = c_P(k)\phi\ \mathfrak{t}_k$ where $t_k \geq 0$ is a function depending on the intensity of the generator's untruthfulness when reporting his production to the Principal and $c_P(k) = c_0 \exp\Big(-\rho\psi(u_P(k))\Big)$ is the cost invested by the Principal per unit of capacity of category k to construct in order to compensate for the missing capacity. It is exponentially decreasing in the Principal's effort.

Proposition 9. *The mechanism m is IR if, and only if when capacity is missing i.e., $\phi < 0$, the Principal's effort toward generator G_k takes the form:*

$$u_P(k) = \psi^{-1}\Big(\frac{1}{\rho}\log\{\frac{c_0\phi\ \mathfrak{t}_k}{(\tilde{p}_k - c_k)\sum_{i=1,\ldots,N} q_{ik} - u_k}\}\Big)$$

Proof of Proposition 9. We determine the values of $u_P(k)$ for which the mechanism

m is IR in the case of missing capacity i.e., $\phi < 0$:

$$\begin{aligned}
\tilde{\pi}_k \geq 0 \quad &\Leftrightarrow \quad (\tilde{p}_k - c_k) \sum_{i=1,\dots,N} q_{ik} - u_k \geq c_P(k)\phi \; \mathfrak{t}_k \\
&\Leftrightarrow \quad \frac{(\tilde{p}_k - c_k) \sum_{i=1,\dots,N} q_{ik} - u_k}{\phi \; \mathfrak{t}_k} \geq c_0 \exp\Big(- \rho\psi(u_P(k))\Big) \\
&\Leftrightarrow \quad \frac{1}{\rho} \log\{ \frac{c_0 \phi \; \mathfrak{t}_k}{(\tilde{p}_k - c_k) \sum_{i=1,\dots,N} q_{ik} - u_k} \} \leq \psi(u_P(k))
\end{aligned}$$

According to the assumptions made on function $\psi(.)$, it is strictly increasing and concave. Therefore, it is bijective. This means that the equation:

$$\frac{1}{\rho} \log\{ \frac{c_0 \phi \; \mathfrak{t}_k}{(\tilde{p}_k - c_k) \sum_{i=1,\dots,N} q_{ik} - u_k} \} = \psi(u_P(k))$$

has a unique solution. This is the one that the Principal will choose since it will minimize his effort while guaranteeing the market opening to new investors. □

Proposition 10. *If the side-payment imposed by the Principal to the generator G_k is of the form:*

$$\mathcal{T}_k = -\mathfrak{K} M_k^* (\hat{q}_k - \bar{q}_k^0)^2$$

then the mechanism m is IC. We set $\mathfrak{K} > 1$ to characterize the strength of the punishment and $M_k^* = \max_{\bar{q}_k \geq 0} \Big\{ \Big[(\tilde{p}_k - c_k) \sum_{i=1,\dots,N} q_{ik} - u_k \Big] - \Big[(\tilde{p}_k|_{\bar{q}_k^0} - c_k|_{\bar{q}_k^0}) \sum_{i=1,\dots,N} q_{ik}|_{\bar{q}_k^0} - u_k|_{\bar{q}_k^0} \Big] \Big\}$; the other agents' capacities being set to the values reported to the Principal.

Proof of Proposition 10. Generator G_k has no incentive to cheat on his capacity production if, and only if:

$$\tilde{\pi}_k|_{\bar{q}_k^0, (\hat{q}_l)_{l\neq k}, (\hat{q}_i^e)_i} > \tilde{\pi}_k|_{\hat{q}_k, (\hat{q}_l)_{l\neq k}, (\hat{q}_i^e)_i}, \; \forall (\hat{q}_l)_{l=1,\dots,\bar{K}} \tag{18}$$

If $\mathcal{T}_k|_{\bar{q}_k^0, (\hat{q}_l)_{l\neq k}} > \mathcal{T}_k|_{\hat{q}_k, (\hat{q}_l)_{l\neq k}} + M_k^*$ then Equation (18) holds, where we set: $M_k^* = \max_{\bar{q}_k \geq 0} \Big\{ \Big[(\tilde{p}_k - c_k) \sum_{i=1,\dots,N} q_{ik} - u_k \Big] - \Big[(\tilde{p}_k|_{\bar{q}_k^0} - c_k|_{\bar{q}_k^0}) \sum_{i=1,\dots,N} q_{ik}|_{\bar{q}_k^0} - u_k|_{\bar{q}_k^0} \Big] \Big\}$. Substituting the side-payment expression in the above inequality, we obtain: $\mathfrak{t}_k|_{\bar{q}_k^0, (\hat{q}_l)_{l\neq k}} < \mathfrak{t}_k|_{\hat{q}_k, (\hat{q}_l)_{l\neq k}} + \frac{M_k^*}{c_P(k)\phi}$. Assuming that the side-payment imposed by the Principal to the generator G_k is quadratic in $\hat{q}_k - \bar{q}_k^0$ i.e., $\mathfrak{t}_k = \bar{k}(\hat{q}_k - \bar{q}_k^0)^2$ with $\bar{k} \geq 0$, we obtain: $-\frac{M_k^*}{c_P(k)\phi} \frac{1}{(\hat{q}_k - \bar{q}_k^0)^2} < \bar{k}$. Energy quantities (produced or reported) are usually large enough to be approximated by integers. Then in the worst case: $-\frac{M_k^*}{c_P(k)\phi} < \bar{k}$. Therefore it is sufficient to choose $\bar{k} = \mathfrak{K}(-\frac{M_k^*}{c_P(k)\phi})$ with $\mathfrak{K} > 1$. □

According to Proposition 10, the side-payment imposed by the Principal to the generator G_k does not depend on the investment performed by the Principal to re-equilibrate the system. But, of course, the strength of punishment can be indexed on this value.

Algorithm 2 Mechanism Design Implementation

1: The capacities $(\bar{q}_k^0)_k$ and $(\bar{q}_i^{e0})_i$ are generated and monitored only by their generator.

2: Generators $(G_k)_k$ and microgrids $(M_i)_i$ report publicly their estimated productions to the Principal.

3: The Principal certifies the capacities using Equations (16) and (17).

4: **Algorithm 1** is run delivering the Nash equilibrium $\Big((\sigma_i^*)_i, (\tilde{\sigma}_k^*)_k\Big)$.

5 a: The true capacity values $(\bar{q}_k^0)_k$ and $(\bar{q}_i^{e0})_i$ are inferred by the Principal from the energy market performance.

5 b: The Principal computes the side-payments $\mathcal{T}_k$ for each generator G_k. If the system lacks the capacity to meet the total demand, the Principal triggers the feedback mechanism: he invests $u_P(k)$ additional units in each generator G_k's equipment and punishes him according to the side-payment $\mathcal{T}_k < 0$.

To compute his optimal decision variables in the Nash equilibrium, each agent takes as input the certified capacities for the other agents and his true hidden capacity value.

4 Numerical illustrations

Energy and capacity market operations are highly dependent on the calibration of the parameters introduced throughout the article. In this section, we first represent numerically the impact of the parameters on the game output, measured through the agents' utilities and social welfare. The paramters should therefore be calibrated depending on the expected output. Second, we study the rate of convergence of the stochastic optimization algorithm used to certify the capacities. Finally, the ability of our mechanism to force the agents to tell the truth is analysed under various punishment strengths.

We model the efficiency of the effort in reducing costs through an s-curve: $\psi(u) = \frac{1}{1+\exp(-\omega u)} - \frac{1}{2}$ where $\omega > 0$ is a parameter characterizing the slope of the s-curve. Indeed, innovation consists in a continual cost reduction resulting from recurring investment on technology upgrades as well as on research and development. Furthermore, the technological progress paths observed in empirical studies identify the performance of a technology with an s-curve [1], [30].

Regarding the parameters, we choose: $\varepsilon = 10^{-5}$, $N = 4, K = 3$ (except for the test of the market structure impact where N and K both span integers between 1 and

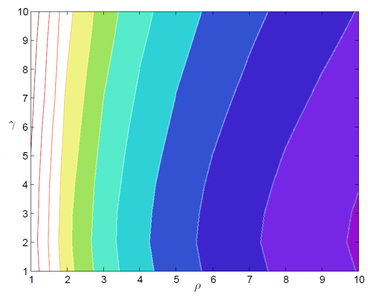
20), $e_1 = 70.67$, $\bar{q}_1^0 = 2.2.10^3$, $\bar{q}_2^0 = 22.10^3$, $\bar{q}_3^0 = 30.10^3$, $D_i \in [2.10^2; 3.10^3], \forall i = 1, 2, 3, 4$, $p_1 = 0.022$, $p_2 = 0.022$, $p_3 = 0.033$, $\beta(2) = 0.53$, $\beta(3) = 0.878$, $\beta(4) = 0.67$, $\theta(2) = 2$, $\theta(3) = 4.8$, $\theta(4) = 2.7$, $c_{k0} \in [10; 100], \forall k = 1, 2, 3$.

4.1 Calibration of the energy market parameters

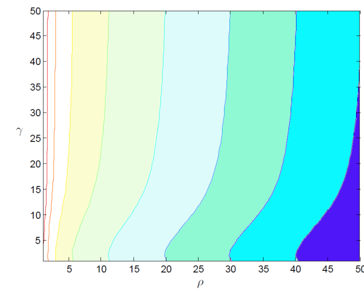
Impact of ρ and γ. We observe in Figure 2 (c) and (d) that the generator's utility is increasing in the technological opportunity and that it reaches its maximum for a substitutability parameter around 8. The decrease of the generator's utility as a function of the increase substituability of the energy sources, as observed in Figure 2 (d), can be explained by the fact that, in case of strong substitutability of the energy sources, the suppliers will choose the cheapest. This price war encourages the generators to align their prices on a common value (cf. Figure 3 (c)), which is smaller than their price under full competition. This phenomenon spreads to the microgrids which are forced to decrease and then to align their demand response prices (cf. Figure 3 (b)). For γ large enough, the supplier will face with a unit price for the generators' productions and a unit price for the demand response ; both of them remaining large (around 0.1 for the generators and around 0.1 for the microgrids). As γ increases, the supplier has less and less opportunity to price discriminate between his energy sources which causes the decrease of his utility (cf. Figure 3 (a)).

Impact of the market structure. We observe, in Figure 4 (a) and (b), that the social welfare is increasing as a function of the competition pressure, both at the supplier and at the generator level, for N and K larger than 8.

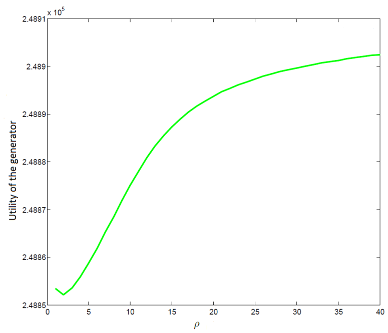
Impact of the disparity of the demand responses performed by the geographically distant microgrids. We evaluate the impact of the disparity of the microgrids' productions on the other agents' utilities. The Gini coefficient can be used to evaluate the disparity of the demand responses. Its value belongs to the interval $[0; 1]$. A value of 0 means perfect equality of the demand responses whereas a value of 1 means total heterogeneity of the demand responses. According to Figure 5 (a), the suppliers have larger utilities when the microgrids' production is rather homogeneous i.e. Gini index smaller than 0.35. Indeed: if all the microgrids produce a high rather homogeneous level of demand response, the suppliers will prefer buying their energy to the microgrid than to the generators whose unit prices are far higher than the microgrids' prices, which remain constant $p_1^e = 2.47, p_2^e = 2.8, p_3^e = 4.7$ (cf. Figure 5 (c)); if all the microgrids produce few, the orders of the suppliers will be so large that the regulator will be forced to intervene. The demand responses becoming more heterogenous (i.e., Gini index larger than 0.35), some suppliers will be forced to buy energy to the generators since, by assumption, there is no communication between the microgrids. To capture more demand, these latter will decrease their price, falling below the demand response prices. In turn, this will enable them to compensate their effort by increasing their revenue (cf. Figures 5 (b) and (d)).



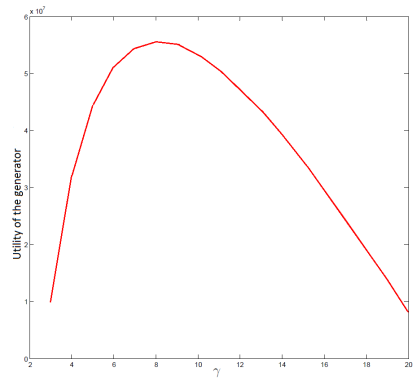
(a)



(b)



(c)



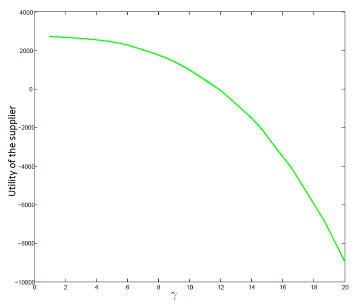
(d)

Figure 2: Evaluation of the impact of the technological opportunity (ρ) and of the substitutability parameter (γ) on the generator's utility while zooming on the parameters i.e., for $\gamma \in [1; 10], \rho \in [1; 10]$ (a), and while dezooming i.e., for $\gamma \in [1; 50], \rho \in [1; 50]$ (b). Making slices of the level sets, we plot the generator's utility as a function of ρ (c) and as function of γ (d).

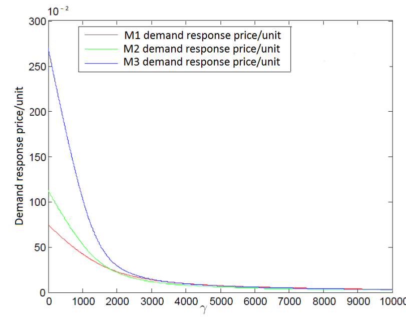
4.2 Convergence of the certification algorithm

The $\bar{K} + N$ nested optimization problems derived from Equations (16) and (17) are solved using the cross-entropy method [28], adapted to optimization. More precisely, we specify a random mechanism to generate feasible solutions (samples) controlled by a set of parametrized probability distribution functions. Based on the performance of the samples, the parameters are updated iteratively by minimizing Kullback-Leibler cross-entropy in order to generate better solutions in next iterations and avoid being trapped in local optima.

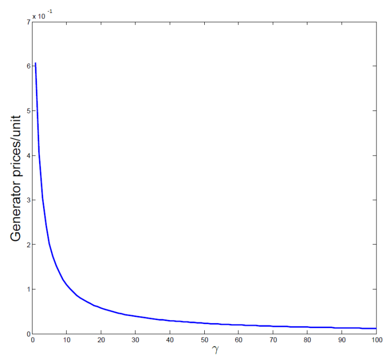
Due to the hierarchical structure of the problem, the trick is to assume that each optimization problem search space is included in the search spaces of agents having highest priority levels. As a consequence, to certify the generator capacity, we have to use the previously certified capacity of the highest priority level generators and similarly for the microgrids. Furthermore, as we saw in the equation (17), the certification



(a)



(b)



(c)

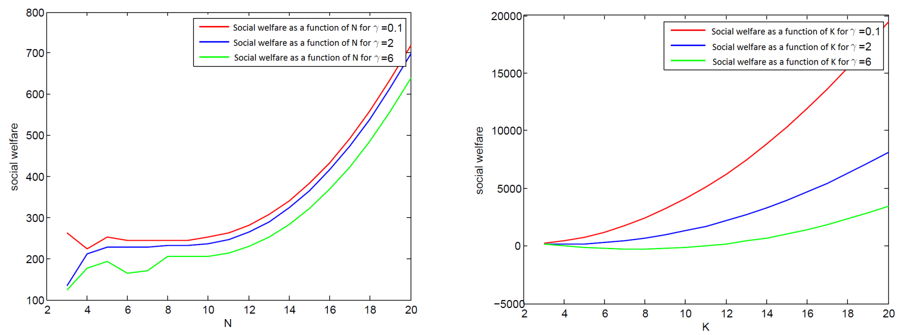
Figure 3: Evaluation of the impact of the substitutability parameter (γ) on the suppliers' utilities (a). In (b) (resp. (c)), we plot the evolution of the microgrids' prices per unit of demand response (resp. the generators's prices per unit of production) as functions of γ.

of the microgrids uses the previously calculated certification of the generators to define the associated optimization problem search spaces. From a computational point of view, this means that we cannot isolate each certification to improve the time of calculation of the certification process but, at the same time, the search spaces being smaller and smaller, this might reduce the rate of convergence. For any $k = 1, ..., \bar{K}$, the computation of the optima M_k^* defined in Proposition 10 is based on the same principle ; the constraints on the capacities being relaxed.

In Figures 6 (a) and (b) resp., the optimal quantities of capacity to certify, for the microgrids and for the generators resp., are represented as functions of the number of iterations required for the stochastic optimization algorithm to converge. We observe that, in both cases, convergence is achieved for a number of iterations smaller than 600.

4.3 To cheat or not to cheat?

We test the effect of bias introduction in the generators' capacity reports, on one of the generators utility, at equilibrium, obtained as output of Algorithm 2, for four values



(a) (b)

Figure 4: Social welfare represented as a function of the competion pressure at the supplier level (a) and of the competition pressure at the generator level (b) for three values of the substitutability parameter i.e., $\gamma = 0.1, 2, 6$.

of the punishment strength: $\mathfrak{K} = 1.2, 3, 6, 10$. We observe in Figure 7 (a) that, for any value of the punishment strength, the more the generator cheats (by increasing the bias in his capacity report), the stronger is the punishment imposed by the Principal. Furthermore, for the considered generator, a threshold effect appears for bias values comprised between 3.5 and 4.5:

- If the bias is smaller or equal to the threshold, the punishment makes the generator's utility decrease but their resulting utility decreasing is inversely proportional to the strength of the punishment. This is due to the heterogenity of the strength of the punishments imposed to the set of generators involved in the game. More precisely, generators having a large strength of punishment $\mathfrak{K}$ are nearly sure to be strongly punished if they cheat. They will therefore prefer small biases which might enable them to increase their revenue while compensating the imposed punishment.

- If the bias is larger than the treshold, the fear of punishment should remove any incentive to cheat.

The suppliers' utilities decrease as the bias in the generators' capacity reports increases (cf. Figure 7 (b)). Indeed, the generators are forced to raise their production prices to compensate the regulator's punishments.

On the overall, the mechanism designed in Section 3 is efficient because it makes the generators' utilities decrease as functions of the bias in their capacity report. But the output result depends on the heterogenity of the strength of the punishments.

5 Conclusion

The originality of this article lies, first, in the coupling of energy and capacity market and, second, in the characterization of their economic behavior through game theory.

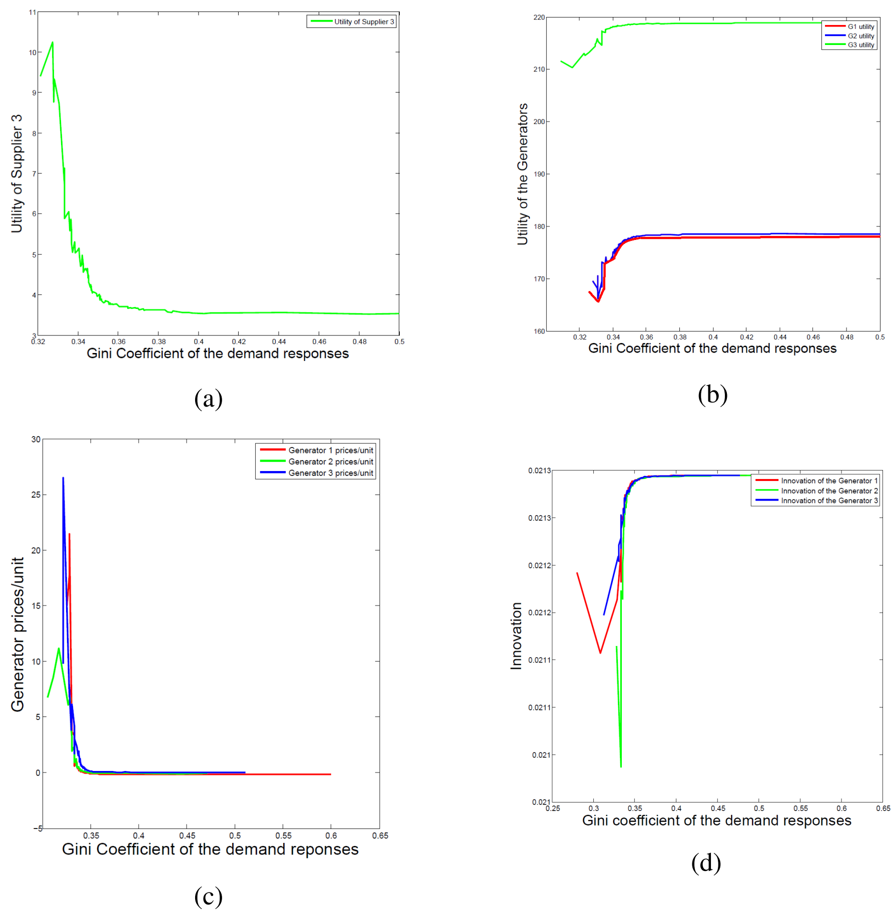
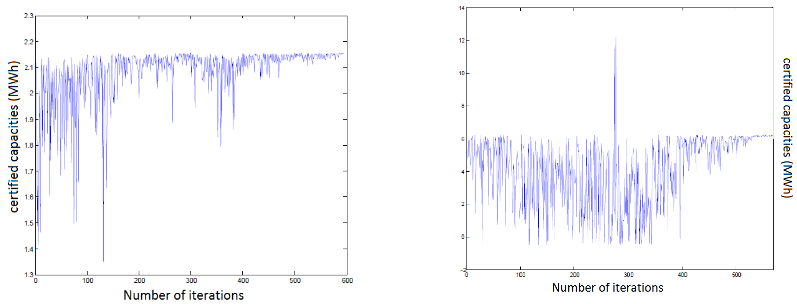


Figure 5: Evaluation of the impact of the disparity of the demand responses on supplier 3's utility (a), on the generators' utilities (b), on the generators' prices (c) and efforts (d).

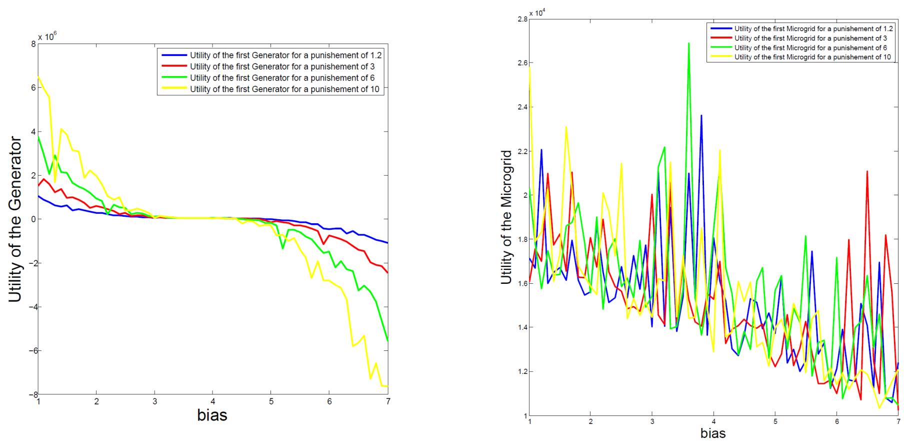
The energy market was represented as a three level decentralized system involving generators, suppliers and microgrids. Our main results can be summarized as follows: we computed analytically the energy market equilibrium in the generators' prices, efforts and in the suppliers' energy orders toward generators and microgrids, which can perform demand response ; we determine analytically the expressions of the price per unit of production for the generators and per unit of demand response, as functions of the price per unit of demand at the consumer level ; finally, we design rules for the capacity market guaranteeing the system wide balance, the market opening to new investors, avoiding moral hazard and abuse of dominant positions. These rules are imposed by the Principal to the generators through punishments in case of system wide imbalance and through the delivery of capacity certificates.

Other economic interpretations can be gained from the analysis of the game output. Tthe normalized coefficients characterizing the energy sources priority might be



(a) (b)

Figure 6: Optimal quantities of capacity to certify for the microgrids (a) and for the generators (b) as functions of the number of iterations in the cross-entropy algorithm.



(a) (b)

Figure 7: Utilities of one generator (a) and of one supplier (b), obtained as output of Algorithm 2. They are functions of the biases introduced by the generators in their capacity reports.

optimized so as to guarantee a minimum level of carbon emissions. Furthermore, it might be used to characterize more finely the economic impact resulting from the introduction of renewable energies and demand response, on the final users' price paid per demand unit and individual welfare.

References

[1] Aghion P., Bloom N., Blundell R., Griffith R., Howitt P., Competition and Innovation: an Inverted U Relationship, Quaterly Journal of Economics, vol.120, pp.701 − −728, 2005

[2] Crampes C., Léautier T.-O., Le Coût des Energies Renouvelables, Le Figaro, 2012

[3] Crampes C., Léautier T.-O., Payer les non-consommateurs d'énergie: une bonne idée dangereuse..., Le Cercle Les Echos, 2011

[4] Cramton P., Stoft S., A Capacity Market that Makes Sense, Electricity Journal, vol.18, pp.$43--54$, 2005

[5] Creti A., Fabra A., Supply security and short-run capacity markets for electricity, Energy Economics, vol.29, pp.$259--276$, 2007

[6] Fabra N., von der Fehr N.-M., de Frutos M.-A., Investment Inventives and Auction Design in Electricity Markets, Discussion Paper, Industrial Organization

[7] Finon D., Defeuilley C., Marty F., Signaux-prix et équilibre de long-terme: Reconsidérer les formes d'organisation sur les marchés électriques, Economie et Prévision: Revue du Ministère de l'Economie et des Finances, 2012

[8] Höffler F., On the consistent use of linear demand systems if not all varieties are available, Economics Bulletin, vol.4, pp.$1--5$, 2008

[9] Jahn A., Beyond Capacity Markets: Delivering Capability Resources to Europe's Decarbonised Power System, RAP: Energy solutions for a changing world, Position Paper, 2012

[10] Jenkins C., Hamilton B., Neme C., Playing with the Big Boys: Energy Efficiency as a Resource in the ISO New England Forward Capacity Market, in proc. ACEEE Summer Study ecoon Energy Efficiency in Building, 2008

[11] Joung M., Baldick R., Kim J., Strategic Behavior in Electricity Capacity Markets, in proc. 42-nd Hawaï International Conference on System Sciences, 2009

[12] Joung M., Kim J., Moral Hazard in the Electricity Capacity Markets, in proc. 14-th International Conference on Intelligent System Applications to Power Systems, 2007

[13] Kind H. J., Nilssen T., Sorgard L., Competition for Viewers and Advertisers in a TV Oligopoly, journal of Media Economics, Taylor and Francis Journals, vol.30, pp.$211--233$, 2007

[14] Léautier T.-O., Maché de capacité électrique: efficacité politique, inefficacité économique, La Tribune, 2012

[15] Le Cadre H., Bedo J.-S., Distributed Learning in Hierarchical Networks, in proc. 6-th International Conference on Performance evaluation Methodologies and Tools, ValueTools 2012

[16] Le Cadre H., Bedo J.-S., Information Management in the Smart Grid: A Learning Game Approach, Working Paper, MINES ParisTech, Centre for Applied Mathematics, 2013

[17] Le Cadre H., Mercier D., Is Energy Storage an Economic Opportunity for the Eco-Neighborhood?, NETNOMICS: Economic Research and Electronic Networking, 2013 DOI:$10.1007/s11066-013-9075-7$

[18] Martin S., Microfoundations for the Linear Demand Product Differentiation Model, with Applications, Working Paper, Purdue University, Department of Economics, 2009

[19] Maneevitjit S., Mount T. D., The Evolution of Capacity Markets in the USA, in proc. 6-th International Conference on the Energy Market, 2009

[20] Marty F., La sécurité de l'approvisionnement électrique: Une nécessaire complémentarité institutionnelle, Working Paper, University of Nice Sophia Antipolis, laboratory GREDEG, 2012

[21] Manez J. A., Moner-Colonques R., Sempere-Monerris J. J., Urbano A., Price differentials among brands in retail distribution product quality and service quality, Working Paper, Catholic University of Louvain, Centre for Operations research and Econometrics, 2011

[22] Mirabel F., The deregulation of electicity and gas market, Presses des MINES, 2012

[23] Myerson R., Game Theory: An Analysis of Conflict, Harvard University Press, 2009

[24] Parkes D. C., Mechanism Design, Harvard University, Ph.D. thesis, 2002

[25] Praktiknjo A., Hähnel A., Erdmann G., Assessing energy supply security: Outage costs in private households, Energy Policy, vol.39, pp.$7825--7833$, 2011

[26] Rious E., Un mécanisme de capacité pour le système électrique français, Le Cercle Les Echos, 2012

[27] Rosellón J., Different Approaches to Supply Adequacy in Electricity Markets, Energy Studies Review, vol.14, article 8, 2006

[28] Rubinstein R. Y., Kroese D. P., The Cross-Entropy Method: A Unified Approach to Combinatorial Optimization, Monte-Carlo Simulation, and Machine Learning, Springer Verlag, 2004

[29] Saad W., Han Z., Poor V. H., Coalitional game theory for cooperative microgrid distribution networks, in proc. of the 2-nd IEEE international workshop on smart grid communications, 2011

[30] Saavedra C., Market structure and technological progress, a differential game approach, in proc. TPRC 2011

[31] Salop S. C., Monopolistic Competition with Outside Goods, The Bell Journal of Economics, vol.10, pp.$141--156$, 1979

[32] Showers M., Demand Response in the US Industrial Sector, Working Paper, Centre for Applied Mathematics, MINES ParisTech, 2012

[33] Skillings S. A., Beyond Capacity Markets - Delivering Capability Resources to Europe's Decarbonised Power System, in proc. of the 9-th International Conference on European Energy Market, 2012

[34] Stoddard R., Adamson S., Comparing Capacity Market and Payment Designs for Ensuring Supply Adequacy, in proc. HICSS Workshop, 2009

[35] Wolak F. A., What's Wrong with Capacity Markets?, Working Paper, Stanford university, Department of Economics, 2004

[36] Nome law, http://www.cre.ft/dossiers/la-loi-nome

[37] Sido-Poignant report, http://www.developpement-durable.gouv.fr/

Contact

Nadia MAÏZI
Directrice du Centre de Mathématiques Appliquées (CMA)

MINES ParisTech / CMA
Rue Claude Daunesse
BP 207
06904 Sophia Antipolis

Tel: +33(0)4 97 15 70 79 / Fax: +33(0)4 97 15 70 66
Courriel: nadia.maizi@mines-paristech.fr

Jean-Charles HOURCADE
Directeur du Centre International de Recherche sur l'Environnement et le Développemenr (CIRED)

CIRED
Campus du Jardin Tropical
45 avenue de la Belle Gabrielle
94736 Nogent sur Marne Cedex

Tel: +33(0)1 43 94 73 63 / Fax: +33(0)1 43 94 73 70
Courriel: hourcade@centre-cired.fr